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Business and Economics

Applying evolutionary game theory to model the division of labour and the dynamics of network

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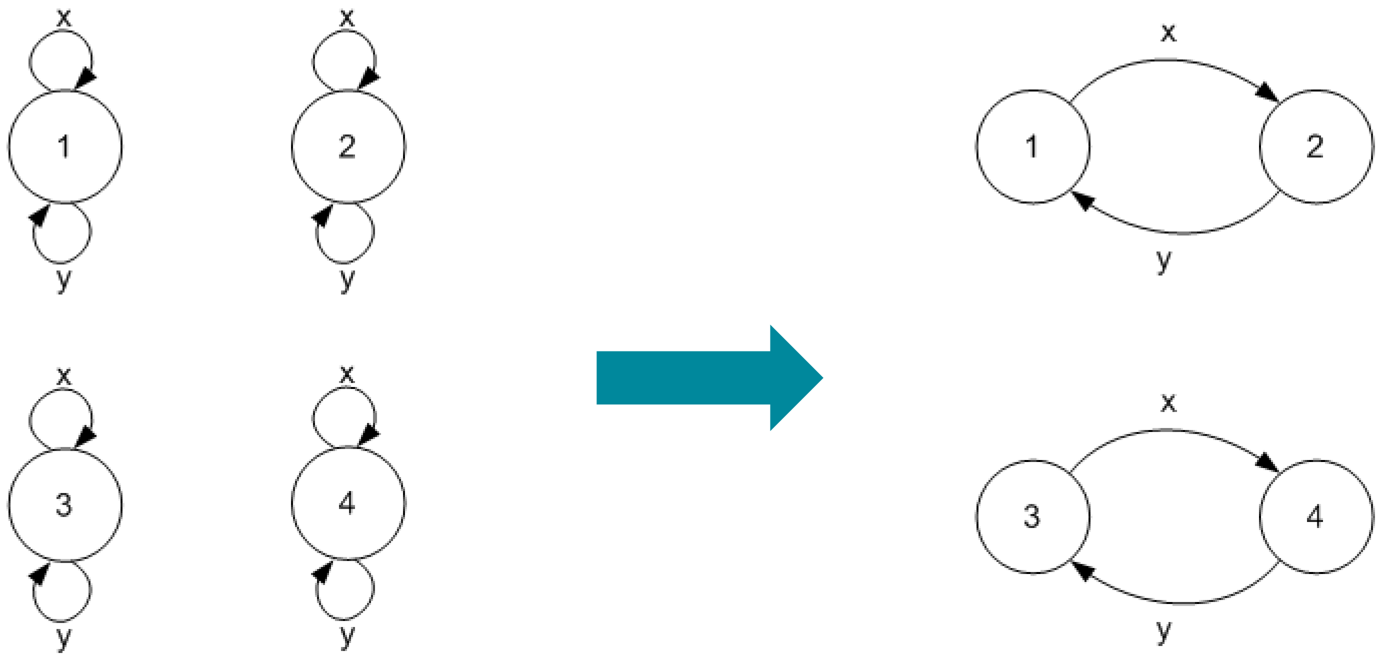
Inframarginal approach – key elements

- Consumer-producer – What to consume and what to produce is a *choice variable*
- Increasing returns to specialization and division of labour (Adam Smith) - Convexity in production at *individual level*
- Transaction costs in exchange (Ronald Coase) – either an *iceberg-type transaction cost* or an endogenized *transaction cost based on information asymmetry*

Inframarginal approach – key elements

- *Corner solutions* – some variables are equal to zero in optimization, which leads to
  - discontinuously jump of economic structures (topological changes)
- *Multiple equilibrium and General equilibrium* – the combination of compatible corner solutions could generate multiple equilibrium; among these, the structure which maximizes *per capita* utility is a general equilibrium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A toy model

- M *ex ante* identical consumer-producers
- 2 commodities (*x* and *y*) – all necessities
- Two alternative exchange patterns with different level of division of labour
  - Autarky: individual self-supplies *x* and *y* – which is technologically inefficient but transactionally (zero) efficient (no coordination problem)
  - Division of labour: some individuals specialize in the production of x and y, respectively and exchange in a market place – which is technologically efficient but transactionally inefficient (e.g. coordination problem)

Specifications

- Utility function: $u = (x + kx^d)(y + ky^d)$
- Production function: $x + x^s = l_x^a,\ y + y^s = l_y^a$, a>1
- Endowment constraint: $l_x + l_y = 1$
- Budget constraint: $x^s + py^s = x^d + py^d$
- Three corner solutions:
  - S_1: Autarky
  - S_2: x-specialist
  - S_3: y-specialist
  - Relative population: $\eta = \{\eta_1, \eta_2, \eta_3\}$

Three corner solutions

Corner solution	Production/Consumption/trade	Indirect Utility
S_1 (Autarky)	$l_x = l_y = \frac{1}{2}, x = y = 2^{-a},$ $x^s = x^d = y^s = y^d = 0$	$u_1(p) = 4^{-a}$
S_2 (x-specialist)	$l_x = 1, l_y = 0, x = x^s = 1/2,$ $y^s = 0, y^d = x^s/p$	$u_2(p) = 4^{-1}kp^{-1}$
S_3 (y-specialist)	$l_x = 0, l_y = 1, y = y^s = 1/2,$ $x^s = 0, x^d = py^s$	$u_3(p) = 4^{-1}kp$

Corner equilibrium

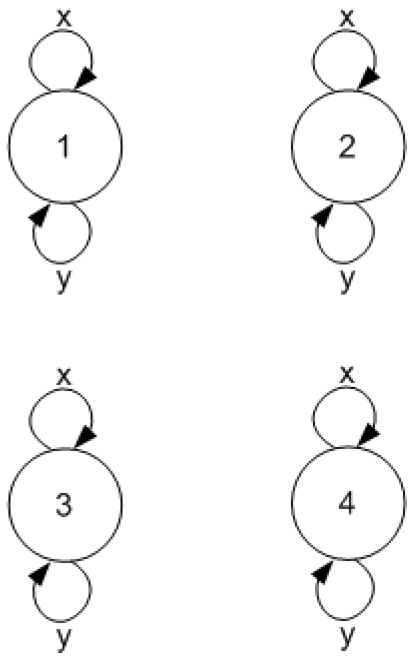
- Utility equalisation:
  - $u_2(p) = u_3(p) \rightarrow p = 1$
  - $u_1(p) = u_2(p) \rightarrow k = 4^{1-a}$
- Market clearing:
  - $\eta_2 M x^s = \eta_3 M x^d \rightarrow \eta_2 = \eta_3$

Static equilibrium and comparative static

- If $k < 4^{1-a}, \mathrm{u}_1 > \mathrm{u}_2 (= \mathrm{u}_3)$, Autarky dominates, so
  - $\eta = \{1,0,0\} \equiv \eta^A$
- If $k > 4^{1-a}, \mathrm{u}_1 < \mathrm{u}_2 (= \mathrm{u}_3)$, Specialisation and division of labour dominates, so
  - $\eta = \left\{0, \frac{1}{2}, \frac{1}{2}\right\} \equiv \eta^{XY}$
- If $k = 4^{1-a}, \mathrm{u}_1 = \mathrm{u}_2 (= \mathrm{u}_3)$
  - $\eta = \alpha\eta^A + (1-\alpha)\eta^{XY}$

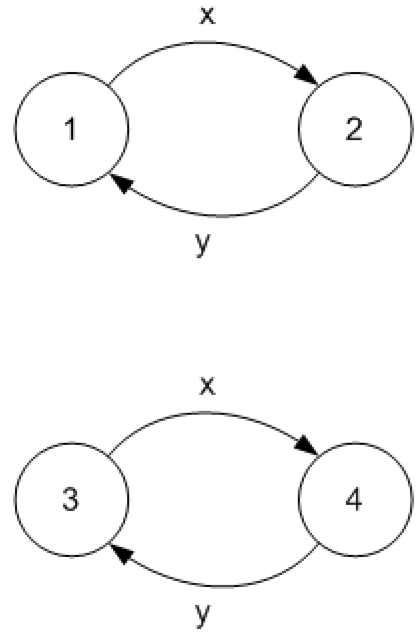
Comparative static

- With an increase in *k*, $\eta^A \rightarrow \eta^{XY}$, means the emergence of
  - The Division of labour
  - Demand/supply and the markets
  - Networks



η^{A} = (1,0,0)

$k > 4^{1-a}$



η^{XY}= (0, ½, ½)

$\eta^A \rightarrow \eta^{XY}$ Dynamics?

- Assume utility equalization does not hold in every instance but market clearing condition continue to hold due to *tâtonnement*.
- Timeline (at time *t*)
  - <u>Morning</u>: choose s_1, or s_2, or $s_{3,}$ lead to:
  $\eta(t) = \{\eta_1(t), \eta_2(t), \eta_3(t)\}$
  - <u>Afternoon</u>: for s_2 and s_3 trade in market place. The market clearing condition:
    $\eta_2(t)Mx^s = \eta_3(t)Mx^d \rightarrow p(t) = \eta_2(t)/\eta_3(t)$
  - <u>Evening</u>: calculate the respective payoff
  $\pi_1(t) = 4^{-a}, \pi_2(t) = 4^{-1}k\eta_3(t)/\eta_2(t), \pi_3(t) = 4^{-1}k\eta_2(t)/\eta_3(t)$

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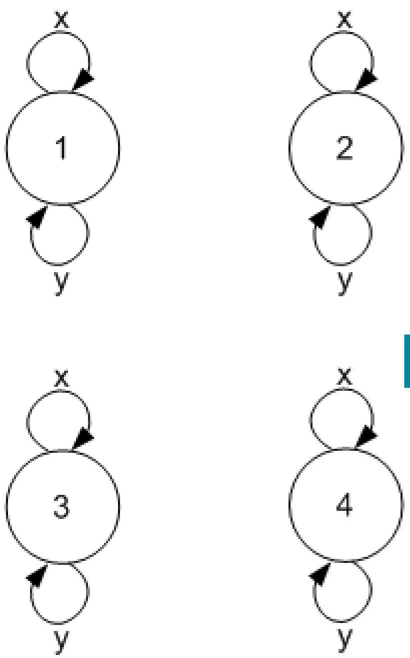
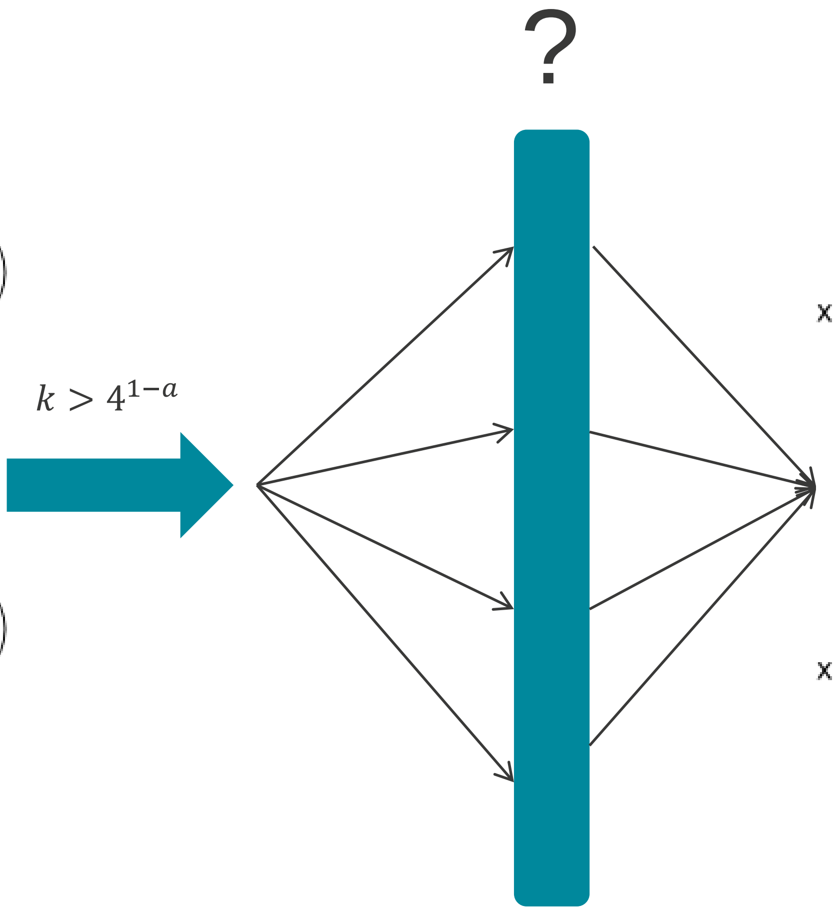
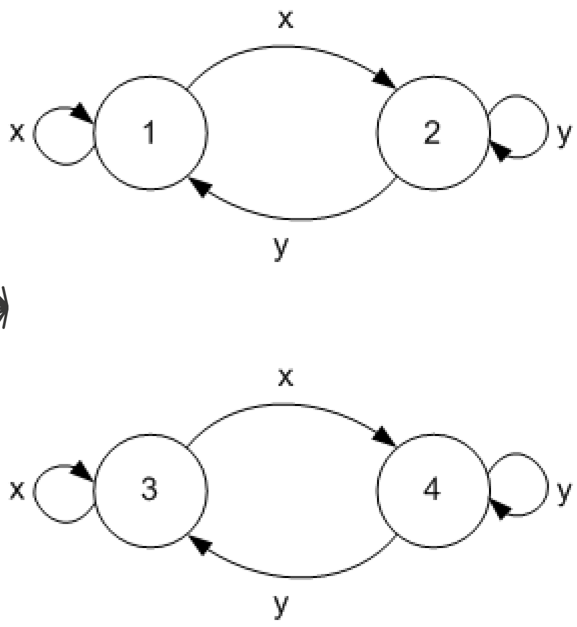


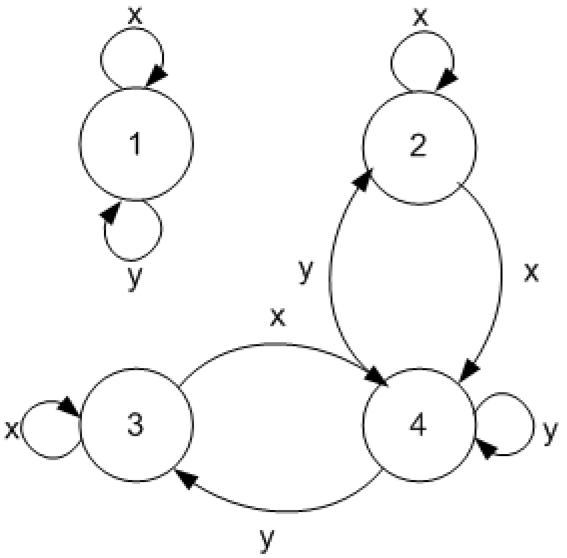
$k > 4^{1-a}$



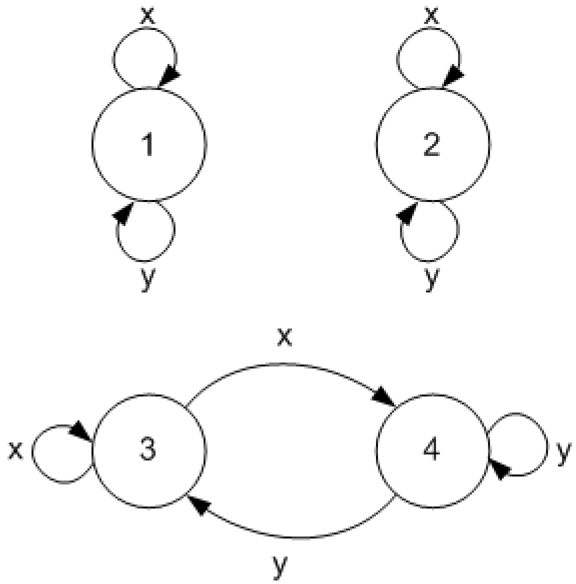


η^{A} = (1,0,0)

η^{XY}= (0, ½, ½)



$$\eta = (\frac{1}{4}, \frac{1}{2}, \frac{1}{4})$$



$$\eta = (\frac{1}{2}, \frac{1}{4}, \frac{1}{4})$$

t+1 adjustment rule

- $\dot{\eta}_j(t+1) = F_j(\pi_1(t), \pi_2(t), \pi_3(t)) = F_j\big(\eta(t)\big),$ $for\ j = 1,2,3$
- In a continuous time, we might just have the following differential equation:

$$\dot{\eta}(t) = F\big(\eta(t)\big)$$

Micro foundation – EGT approach

- Assume the whole population can be divided into two groups:
  - Imitators: majority of the population who will revise their strategies according to the comparison of their current payoffs with reference payoffs
  - Innovators: minority of the population who will look at the potential payoffs of various strategies and choose autarky if k$\leq 4^{1-a}$, and choose x- or y-specialist with equal probability if k$> 4^{1-a}$

More on imitator

- An imitator will compare her payoff π of the current strategy *s* with the payoff π' of a reference strategy *s'*, which is the strategy adopted by a person that the imitator randomly met in the population, to determine whether it is worth changing
- An adjustment cost *c* must be paid instantaneously when the imitator decides to change her strategy, where the adjustment cost is a random variable that is independently determined across individuals and time by the uniform probability distribution over $[0,\bar{c}]$.
- So, an imitator will change her strategy if $c \leq \pi' - \pi$

Replicator Dynamics (RD)

- If all imitator adopt such a strategy, the adjustment process must coincide with Replicator Dynamics (Vega-Redondo, 1996)

$$\dot{\eta}_j(t) = \frac{1}{\bar{c}}\eta_j(t)\left[\pi_j\big(\eta(t)\big) - \bar{\pi}\big(\eta(t)\big)\right]; j = 1,2,3$$

Where $\bar{\pi}\big(\eta(t)\big)$ is the average payoff of three strategies

- For simplicity, assume $\bar{c} = 1$

More on innovator

- An innovator will adopt strategies s_1, s_2, and s_3 with the probability of $\varepsilon, \frac{1-\varepsilon}{2}, \frac{1-\varepsilon}{2}$, where

$$\varepsilon = \begin{Bmatrix} 0, 4^{1-a} < k \\ 1, 4^{1-a} \geq k \end{Bmatrix}$$

Imitator + innovator

- Assume r is the percentage of innovator, so the dynamics of η could be expressed as:

$$\dot{\eta}_1(t) = (1-r)\eta_1(t)\left[\pi_1\left(\eta(t)\right) - \bar{\pi}\left(\eta(t)\right)\right] + r[\varepsilon - \eta_1(t)]$$

$$\dot{\eta}_2(t) = (1-r)\eta_2(t)\left[\pi_2\left(\eta(t)\right) - \bar{\pi}\left(\eta(t)\right)\right] + r[(1-\varepsilon)/2 - \eta_2(t)]$$

$$\dot{\eta}_3(t) = (1-r)\eta_3(t)\left[\pi_3\left(\eta(t)\right) - \bar{\pi}\left(\eta(t)\right)\right] + r[(1-\varepsilon)/2 - \eta_3(t)]$$

Proposition

- It can be approved that as long as

$$r > \frac{4^{-a}}{1 + 4^{-a}}$$

for any $\eta(0)$, the economy converges at η^{A} if k< 4^{1-a}, converges at η^{XY}if $k < 4^{1-a}$, converges at $\alpha\eta^{A} + (1 - \alpha)\eta^{XY}$when $k = 4^{1-a}$.

- This is the evolutionary stable state (ESS)

Interpretation of the *r* condition

- Even if $k > 4^{1-a}$, if *r* is not sufficiently large, say *r*=0, *then* $\dot{\eta}_1(t) = \eta_1(t)\left[\pi_1\big(\eta(t)\big) - \bar{\pi}\big(\eta(t)\big)\right]$. For some η$(0)$, $\pi_1\big(\eta(t)\big) - \bar{\pi}\big(\eta(t)\big)$>0, so $\dot{\eta}_1(t)$>0, so the economy might converge to η^A, rather than η^{XY}
- Intuitively, if $\eta_2(0) \gg \eta_3(0)$, p - price of y will be very high and $\pi_2(0)$ will be very low.
  - The autarky will have higher probability to meet a x-specialist and will have no motivation to change.
  - x-specialist will see the higher payoff of the autarky, and the imitator will change to be an autarky. Consequently, $\dot{\eta}_1(t)$>0 and η^A dominates

Interpretation of the *r* condition

- An innovator, however, insists in becoming a specialist as long as $k > 4^{1-a}$ - which makes , $\dot{\eta}_1(t)$ <0 and η^{XY} dominates
- This highlights the importance of innovator in promoting the evolution of economic structure – in a Schumpeterian fashion.