Stability and Fragility: The Dynamics of General Equilibrium

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Summary

The standard macroeconomic model used by economic policy planners and central bank planners cannot handle questions of fragility and stability because

the issue involves dynamical systems theory, while the standard macro models (DSGE) are comparative static, and

stability depends on within-sector firm interdependencies, while the DSGE models assume one firm per sector (and usually one sector only).

Summary

A successful alternative to DSGE must handle the traditional macroeconomic phenomena (unemployment, inflation, debt, terms of trade) as well as the dynamical issues of stability and fragility.

To be legitimate for economic policy, a politically viable alternative must be accepted by professional economists, as measured by its success in appearing in the leading economic journals.

Failure to penetrate the economics discipline would be our failure, not theirs: blame the victim---make no excuses!

To accomplish this, we need a general theory of market dynamics, of which financial markets are a special case, where the specific characteristics of financial markets account for their increased fragility and vulnerability.

Summary

A successful alternative will abandon rational expectations theory, and even abandon modeling expectations altogether, in favor of modeling individual adaptive and imitative behavior, perhaps augmented by a theory of how minds "network" in evolving collective attitudes and behaviors.

Rational expectations are relevant for simple dynamics (e.g., playing a card game), but not for complex dynamics.

What is General Equilibrium?

There is one generally accepted model of the large-scale behavior of the market economy, known as Walrasian general equilibrium.

The Swiss economist Léon Walras created this theory in 1874-1877 in his Elements of Pure Economics

Léon Walras, 1834-1910

What is General Equilibrium?

The Walrasian economy consists of households and firms.

Firms buy or rent the services of inputs at given market prices,

combine them to produce outputs

which they sell at given market prices to the households.

What is General Equilibrium?

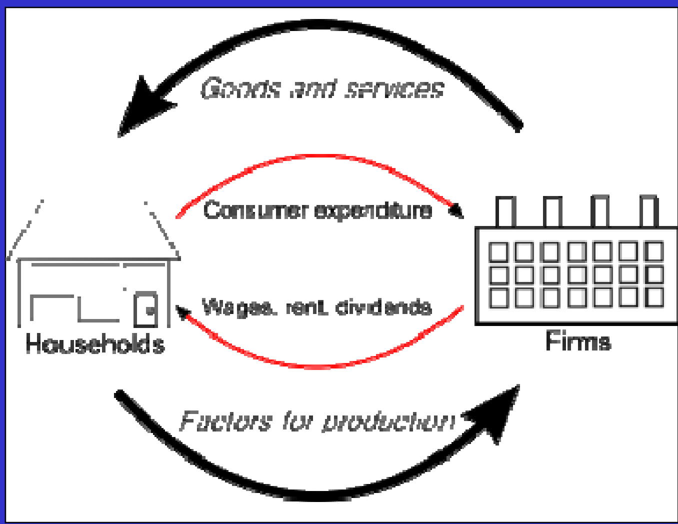
Inputs include labor, capital goods (rented), raw materials, and the outputs of other firms (purchased).

Inputs, as well as shares in the net profit of the firms, are owned by the households, and form their wealth.

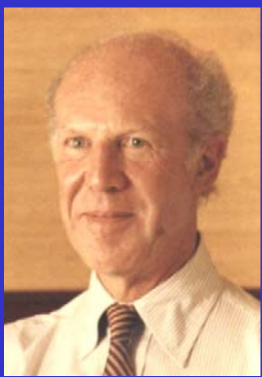
In each period, households buy the output of the various firms, some of which they consume, and some of which they add to their stock of wealth.

What is General Equilibrium?

The economy is in equilibrium when the prices p are set so that supply = demand in each sector (a sector is a set of firms that all sell the same good).



What is General Equilibrium?



Gerard Debreu, 1921-2004

In the period 1952-1954, Kenneth Arrow and Gerard Debreu showed that with plausible assumptions, there exists a set of equilibrium (market clearing) prices.

Kenneth Arrow, 1921-

The Quest for Stability

The question of stability of the Walrasian economy was a central research focus in the years immediately following the existence proofs

(Arrow and Hurwicz, 1958, 1959, 1960; Arrow, Block and Hurwicz, 1959; Nikaido 1959; McKenzie, 1960; Nikaido and Uzawa 1960; Uzawa 1960a,b).

Their models assumed that out of equilibrium, there is a system of public prices shared by all agents,

the time rate of change of prices being a function of aggregate excess demand.

The Quest for Stability

The public price system was implemented by a single agent (the auctioneer)

acting outside the economy to update prices in the current period on the basis of the current pattern of excess demand,

using a process of “tâtonnement,” as was first suggested by Walras himself.

Walras' Faute de Mieux Auctioneer

The auctioneer, before any buying and selling takes place,

1. "calls out" a set of prices,
2. asks firms and households say how much they want to buy and sell at these prices,
3. calculates the "excess demand" or "excess supply" for each sector,
4. adjusts the prices to bring the markets closer to equilibrium,
5. Then back to 1, until all markets are in equilibrium.
6. **Only then** is production and trade permitted, at the specified market-clearing prices.

The Quest for Stability

The quest for a general stability theorem was derailed by Herbert Scarf"s simple examples of unstable Walrasian equilibria (1960).

There were attempts soon after to continue the analysis of tâtonnement by adding trading out of equilibrium (Uzawa 1959, 1961, 1962; Negishi 1961; Hahn 1962; and Hahn and Negishi 1962), but with only limited success.

The Quest for Stability

General equilibrium theorists in the early 1970's harbored some expectation that plausible restrictions on the shape of the excess demand functions might entail stability, but Sonnenschein (1973), Mantel (1974, 1976), and Debreu (1974) showed that any continuous function, homogeneous degree zero in prices, and satisfying Walras' Law, is the excess demand function for some Walrasian economy.

The Quest for Stability

Hahn and Negishi (1962) showed that if out-of-equilibrium trade is permitted and the so-called Hahn condition obtains, then the Walrasian equilibrium is stable under tâtonnement.

The Hahn condition says that if there is aggregate excess demand then no individual experiences excess supply, and if there is aggregate excess supply, then no individual experiences excess demand. Fisher (1983) significantly broadened this model.

The Quest for Stability

Surveying the state of the art some quarter century after Arrow and Debreu's seminal existence theorems, Fisher (1983) concluded that little progress had been made towards an acceptable model of Walrasian market dynamics.

In fact, the tâtonnement process generically implies chaotic price dynamics (Saari 1985, Bala and Majumdar92).

Saari (1995) and others have shown that the information needed by tâtonnement include virtually complete knowledge of all cross-elasticities of demand in addition to excess demand quantities.

The Quest for Stability

It is now more than another quarter century since Fisher's remarks, but it remains the case that we know nothing systematic about Walrasian market dynamics in realistic economies

Modern Macroeconomic Theory

Modern macroeconomics rejects the general equilibrium model in favor of highly aggregated `toy' models of the economy.

Keynesian theory assumes one good with two uses (consumption and investment)

and two prices: the wage rate and the interest rate.

The wage rate is rigid downwards because workers will not accept wage cuts,

and the interest rate does not clear the savings/loans market because of liquidity preference.

Modern Macroeconomic Ideology

Chicago-school macroeconomics rejects the `irrationalities' of the Keynesian model in favor of
`rational expectations',
retaining the toy economy assumptions,
to which are added
stable and continuously clearing markets.

Modern Macroeconomic Ideology

Keynesian economists George Akerlof and Bradley Schiller's ***Animal Spirits*** (Princeton, 2009) has become the rallying-cry for the reassertion of the importance of state regulation and control of the market economy. They say:

"if we thought that people were totally rational…

we too would believe that government should play little role in the regulation of financial markets, and perhaps even in determining the level of aggregate demand." (p. 173).

Modern Macroeconomic Ideology

Yet there is nothing in economic theory, and no empirical evidence, that markets are intrinsically stable and resilient in the face of macro-level shocks.

A Markov Process Primer

- Consider an economy with k goods and n agents.
- In each period, each agent accepts one good as money.
- In each period, one agent switches from his own preferred money to that of his randomly encountered trading partner.
- We can describe the state of the economy as a k-vector $(w1,\ldots,wk)$, where wi is the number of agents who accept good i as money.
- The number of states in the economy is $C(n+k\text{-}1,k\text{-}1)$, where $C(n, k) = n!/(n-k)!k!$.

A Markov Process Primer

For instance, if n=100 and k=10, then the number of states S in the system is S = C(109,9) = 4,263,421,511,271.

The matrix of probabilities P of moving from one state to another in a single period is called the probability transition matrix of the system, and the system is called a Markov process.

This matrix has 18176762982768297580035441 entries (about 1.8 E+25).

A Markov Process Primer

What is the long-run behavior of this Markov process?

If we start in state *i* at time $t = 0$, the probability $p_{ij}(2)$ of being in state *j* at time $t=2$ is just the *ij* th entry in the matrix $P \times P = P^2$.

Thus to calculate the two-period transition matrix, we require about 1.0E+51 multiplications and additions.

There is a perfectly determinate way of calculating the long-run behavior of the Markov process, which is the limit of P^n as n goes to infinity.

A Markov Process Primer

However, it is possible to describe the behavior of this Markov process quite simply.

When the last person to use a good as money switches to another good, that good will never reappear as a money good for any agent.

There is a positive probability that any good that is not used by all agents as money will go extinct within *n* periods.

Thus with probability one, all goods but one will go extinct.

A Markov Process Primer

The *k* states in which all agents use the same good as money are called *absorbing states*.

All other states are *transient*, which means that with probability one they appear only a finite number of times.

The general Markov process is just like this, except that the absorbing states are replaced by more complex *Markov subprocesses* called ergodic Markov processes.

From Differential Equations to Markov Processes

An appropriate candidate for modeling the Walrasian system in disequilibrium is a Markov process.

The states of the process are vectors whose components are the states of individual agents.

The state of each agent includes his holding of each good, an array of parameters representing his search strategies for buying and selling, parameters representing his linkage to others in a network of traders, and finally his vector of private prices, which the agent uses to evaluate trading offers.

Markovian Market Dynamics

If state *i* has a positive probability of making a transition to state *j* in a finite number of periods, we say *I* communicates with *j*.

If all states in a Markov process communicate with each other, we say the process is irreducible.

We cannot assume a Markovian market economy is irreducible.

Markovian Market Dynamics

The Markovian market model is finite if we assume there are a finite number of agents, a finite number of goods, a minimum discernible quantity of each good, and a finite inventory capacity for each good.

A strictly positive probability of remaining in the same state for an agent then ensures that the Markov process is aperiodic.

An aperiodic, irreducible, and finite Markov process is ergodic (Feller 1950),

which means has a stationary distribution expressing the long-run probability of being in each state of the system, irrespective of its initial state.

Markovian Market Dynamics

THE ERGODIC THEOREM: Consider an n-state aperiodic and irreducible Markov process with transition matrix P, so the t-period transition probabilities are given by $P^{(t)} = P^t$. Then there is probability distribution $\pi = (\pi_1, \ldots, \pi_n)$ over the states of the Markov process with strictly positive entries that has the following properties for $j = 1, \ldots, n$:

$$\pi_j = \lim_{t \to \infty} p_{ij}^{(t)} \quad \text{for } i = 1, \ldots, n \tag{2}$$

$$\pi_j = \sum_{i=1}^{n} \pi_i p_{ij} \tag{3}$$

We call π the *stationary distribution* of the Markov process.

Markovian Market Dynamics

A Markov process has only one period "memory," but it is easy to show that this extends to an L-period memory for any finite L.

It is easy to aggregate the states of a Markov process if it suits us (e.g., to talk about aggregate demand, or average price),

and under suitable conditions, the aggregates also form a Markov process.

The Structure of Aperiodic Finite Markov Processes

If a Markov process is finite and aperiodic but is not irreducible, its states can be partitioned into subsets $S^{\natural}, S_1, \ldots S_k$, where *every* state $s \in S^{\natural}$ is *transient*, meaning that for any realization $\{s^t\}$ of the Markov process, with probability one there is a time t such that $s \neq s^{t+t'}$ for all $t' = 1, 2, \ldots$; i.e., s does not reappear in $\{s^t\}$ after time t. It follows that also with probability 1 there is a time t such that no member of $S^{\natural}$ appears after time t. A non-transient state is called *recurrent*, for it reappears infinitely often with probability one in a realization of the Markov process.

The Properties of Markovian Economies

There are well-known and elegant ways of finding the stationary distribution of small Markov processes (Gintis, Game Theory Evolving, 2009),

but the Markovian economy, even with only a few thousand agents, has more states than there are atoms in the universe.

Thus, even though the stationary distribution of a Markovian economy has an explicit analytical solution,

the solution is so large that we cannot even write it down.

The Properties of Markovian Economies

Suppose an aperiodic Markov process M with transient states St and ergodic subprocesses, S1,…,Sk enters a subprocess Sr after t0 periods with high probability, and

suppose the historical average over states form t0 to t1 is a close approximation to the stationary distribution of Sr. Consider the Markov process M+ consisting of reinitializing M every t1 periods.

The M+ is ergodic, and a sufficiently large sample of historical averages starting at t0 periods after reinitialization will reveal the stationary distribution of M+.

This is the methodology we will use in estimating the aggregate properties of a Markov model of a market economy.

A Decentralized Market System with Individual Production

- We model a decentralized market economy with many goods, each agent producing one good and consuming many.
- Agents congregate at markets for their production good and periodically visit markets for goods they consume.
- Agents have private prices, and an exchange takes place when it benefits both parties according to their private prices.
- Agents adjust their prices according to experience, and they periodically copy the price structure of another agent who has been more successful than himself.

A Decentralized Market System with Individual Production

We find that

- the economy moves quickly from private to quasi-public prices,
- the latter being private prices with low standard error across agents, and
- in the long run, quasi-public prices move to general Walrasian quasi-equilibrium,
- which is a stationary distribution with near-market-clearing prices in almost all periods.

Private to Quasi-Public Prices

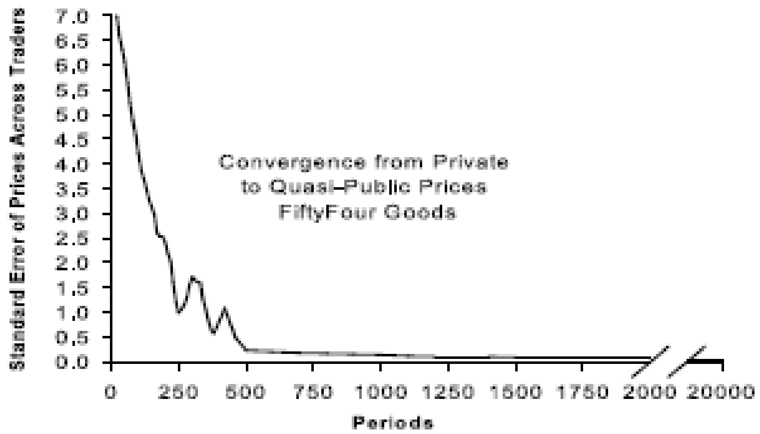


Figure 4: Convergence of private prices to quasi-public prices in a typical run with nine goods in 6 styles each (fifty-four goods).

Quasi-Market Clearing

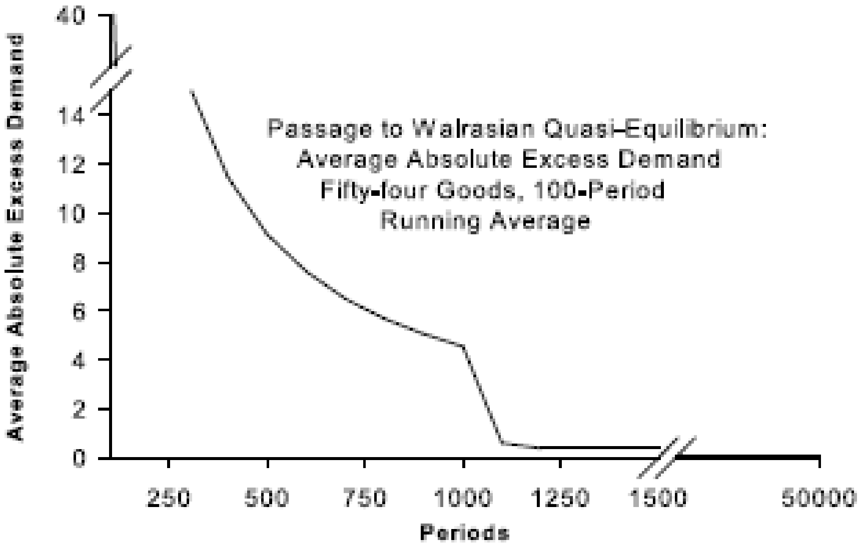
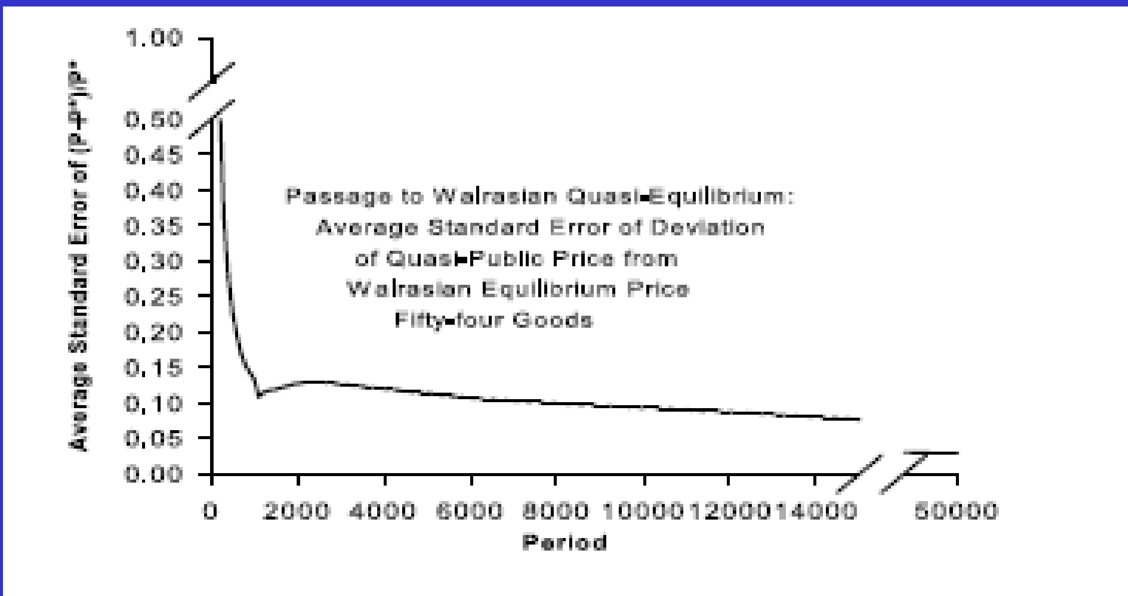


Figure 5: The path of aggregate excess demand over 50,000 periods.

Quasi-Public Prices to Walrasian Equilibrium Prices



Spontaneous Emergence of Money in the Markovian Market Economy

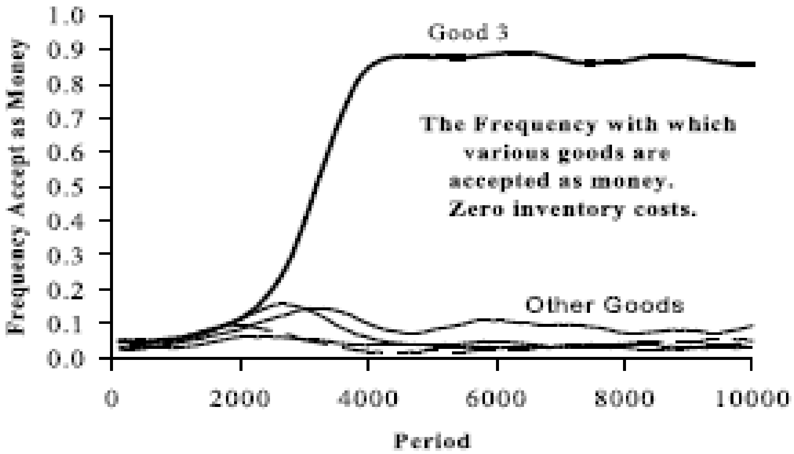


Figure 7: The Emergence of Money in a Market Economy. The parameters of the model are the same as in the baseline case treated previously. Inventory costs are assume absent.

Resilience and Fragility of the Markovian Market Economy

Consider a Markovian market economy with *fiat* money (medium of exchange that is not consumed or produced).

We impose an aggregate shock on the economy by halving the money holdings of each agent every 1000 periods, then restoring the money supply after an addition 100 periods have passed.

Resilience and Fragility of the Markovian Market Economy

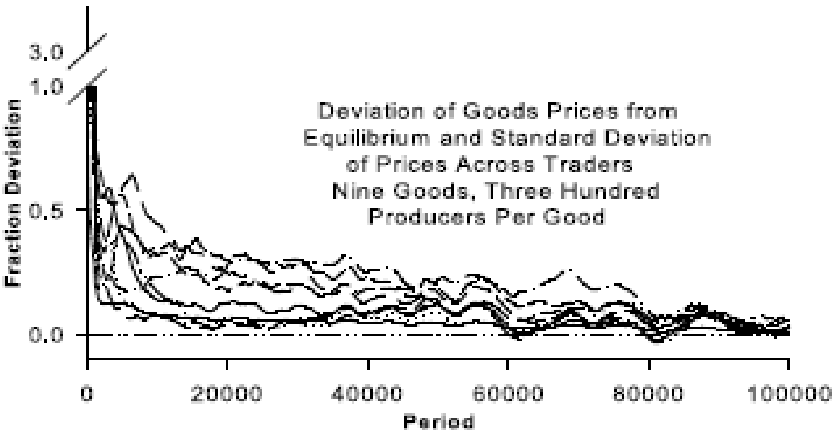
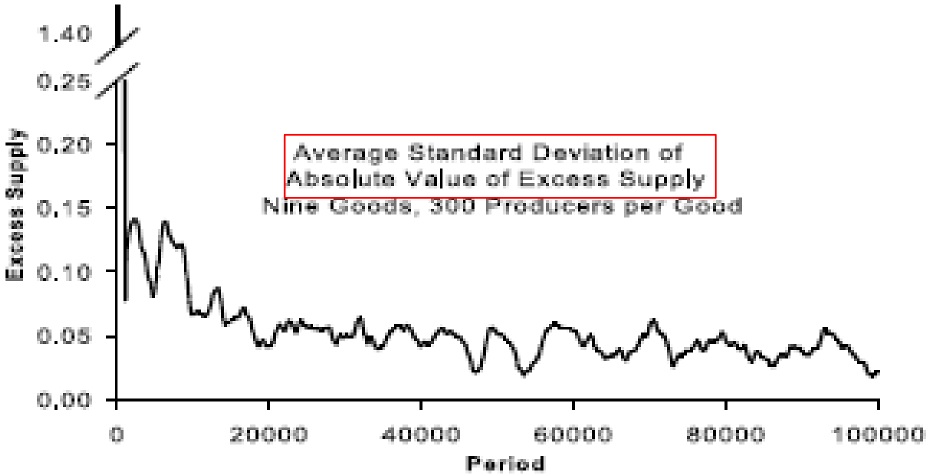


Figure 9: Walrasian equilibrium is resilient to aggregate shocks. With 300 producers per good, we impose an aggregate shock on the Markov process consisting of halving the money supply every 1000 periods, and restoring the money supply after 100 periods have elapsed with the smaller money supply. There is virtually no effect on the passage to a quasi-Walrasian equilibrium.

Resilience and Fragility of the Markovian Market Economy



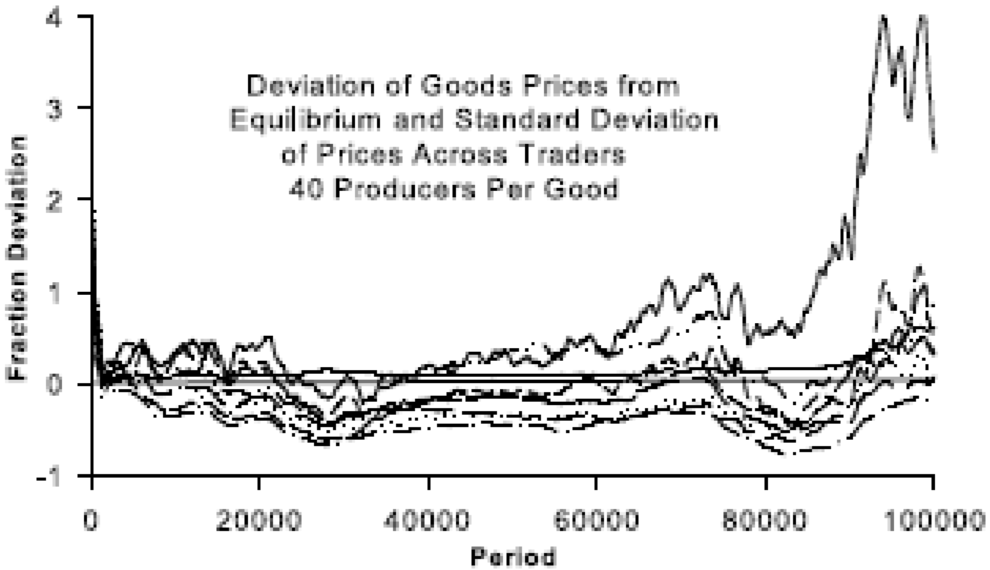
Note that there us excess demand volitility, but not fragility.

Figure 10: Excess demand is resilient in the face of large macro-level shocks. The parameters are as in the previous figure. Note that there is virtually no effect on aggregate excess demand in any sector of the economy.

Resilience and Fragility of the Markovian Market Economy

Now we reduce the number of producers per sector from 300 to 40. The same shock pattern produces instability.

Resilience and Fragility of the Markovian Market Economy



Resilience and Fragility of the Markovian Market Economy